Identifying Model-Based Reconfiguration Goals through Functional Deficiencies

Emmanuel Benazera[1] and **Louise Travé-Massuyès**[2]

Abstract. Model-based diagnosis is now advanced to the point autonomous systems face some uncertain and faulty situations with success by correctly identifying sets of potential component faults. The next step toward more autonomy is to have the system recovering itself after faults occur, a process known as *model-based reconfiguration*. After faults occur, given a prediction of what the nominal expected state of the system should be and the belief state that results from the diagnosis operation, this paper shows how to automatically determine the *functional deficiencies* induced by the faults. These deficiencies are characterized in the case of uncertain state estimates. A methodology is then presented to determine the reconfiguration goals from the deficiencies. Finally, a recovery process interleaves planning and model predictive control to restore the lost functionalities in prioritized order.

1 Introduction

Model-based autonomous systems already face faulty situations with some success: they detect and diagnose faults by either identifying potential candidates for their own physical state [6] or reasoning on their structural and behavioral knowledge [5]. The next step toward more autonomy is to have the system recovering itself after faults occur, a process known as *model-based reconfiguration*[3] (MBReconf). Automated reconfiguration comprehends three steps: goal identification, goal selection, recovery. *Goal identification* searches for a set of potential states of the system where the fault effects are inhibited; *goal selection* is the process of deciding the best of these states, denoted goal states; *recovery* searches for the chain of actions that may turn the physical system state into the desired goal states. Recent architecture design for autonomy [10] puts the goal identification and selection processes outside the scope of a model-based diagnoser, in the hands of upper decisional levels. The aim of this paper is to produce an automated goal identification/selection/recovery methodology that takes better advantage of the system model. Due to several factors, MBReconf is a challenging problem:

- The state of the system cannot be uniquely determined in all situations. Recent model-based monitoring/diagnosis systems track several potential non-faulty/faulty state estimates simultaneously [11, 2]. Moreover, the set of state estimates is the result of a selection process as the total number of possible states is too large to be explored. The ambiguity is however mitigated by the fact that the number of state estimates is typically small.
- Faults effects may differ from one state estimate to the other. For this reason, pre-compiled policies may fail recovering the system by proposing an improper command when the state is uncertain.
- Nowadays, embedded digitally controlled systems have complex behaviors characterized by a preeminence of discrete switches in their dynamics. They are modeled as hybrid systems, that exhibit both discrete and continuous dynamics.

Referring to the *faulty states* as the estimates that result from the diagnosis operation, as opposed to the nominally *predicted states*, we propose to compare the faulty states and the predicted states to determine the *functional deficiencies* caused by the faults. In this context, functional deficiencies are variable instances in one or more predicted states and that have been *lost* in one or more faulty states. Our approach aims at minimizing the size of a functionality to recover while maximizing its coverage of the estimates. The contributions of this paper are threefold. First, we show how this strategy leads to a finite set of disjoint functional deficiencies, and characterize them. Second, we propose a methodology to identify potential goals from the deficiencies based on a productive analogy with model-based diagnosis, reasoning at a single point in time, despite the system continuous dynamics. Third, we show how to interleave conformant planning and model predictive control to bring the system's hybrid dynamics from the initial faulty (uncertain) state to the potential goal state.

2 Hybrid Model-Based State Prediction and Diagnosis

In this section we introduce a comprehensive formalization of model, state and uncertainty. The autonomous system is considered a model-based system, i.e. that has a structural and behavioral knowledge of itself.

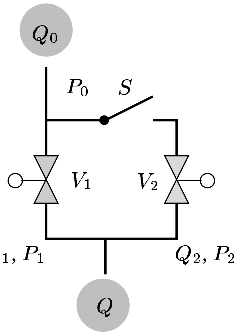
Definition 1 (Model-Based System). *A model-based system A is a tuple $(\mathcal{C}, \mathcal{M}, \mathcal{T}, \mathcal{X}, E)$, where $\mathcal{C}$ is a set of modeled components, $\mathcal{M}$ a set of finite discrete variables as component behavioral modes, $\mathcal{T}$ a set of transitions among these modes, $\mathcal{X}$ the set of continuous variables partitionned in state variables $\mathcal{X}_X$, output (observed) variables $\mathcal{X}_Y$ and input variables (commands) $\mathcal{X}_U$. E is a set of continuous static/differential equations over $\mathcal{X}$.*

In this paper we use a hybrid description of the physical system's state. The *hybrid state* s is the tuple (M, X). Instances of variables v in $M \cup X$ are noted $(v = v^j)$, or v^j for short. The hybrid state's discrete side abstracts the physical system as a set of mode instances $M = \bigwedge_k C_k.m^{i_k}$ where $C_k.m^{i_k}$ is an instance of a variable $m \in \mathcal{M}$ of component $C_k \in \mathcal{C}$. The continuous state X is

[1] RIACS/NASA Ames Research Center, Moffett Field, California 94035 email: ebenazer@email.arc.nasa.gov

[2] LAAS-CNRS, 7, av. du Colonel Roche, 31077 Toulouse Cedex 4 email: louise@laas.fr

[3] For now, most embedded controllers include pre-compiled recovery policies as part of a rule-based system.



$$V_i \left\{ \begin{array}{l} Q_i = k_i S_i \sqrt{P_0 - P_i} \textbf{ if } \phi_i \\ Q_i = 0 \textbf{ if } \neg\phi_i \\ \phi_i\colon P_0 \geq P_i \\ \quad \wedge (V_i.m = open \vee V_i.m = stuck\_open) \\ \\ \tau_{11}\colon V_1.m = closed \wedge V_1.cmd = op \rightarrow V_1.m = open \\ \tau_{12}\colon V_1.m = open \wedge V_1.cmd = cl \rightarrow V_1.m = closed \\ \tau_{21}\colon V_2.m = closed \wedge \textbf{S.m=closed} \rightarrow V_2.m = open \\ \tau_{22}\colon V_2.m = open \wedge \textbf{S.m=open} \rightarrow V_2.m = closed \end{array} \right.$$

$$S \left\{ \begin{array}{l} \tau_3\colon S.m = open \wedge \mathbf{(}P_0 \geq P^*\mathbf{)} \rightarrow S.m = closed \\ \tau_4\colon S.m = closed \wedge \mathbf{(}P_0 < P^*\mathbf{)} \rightarrow S.m = open \end{array} \right.$$

Input connection: $Q_0 = Q_1 + Q_2$

$$\text{Output connection: } \left\{ \begin{array}{l} Q = Q_1 + Q_2 \\ P_1 = P_{atm} \\ P_2 = P_{atm} \end{array} \right.$$

Figure 1. Pressure expansion system

made of instances x^j of continuous variables of $\mathcal{X}_X$. Observed variables of $\mathcal{X}_Y$ are noted y (vector Y), and $\tilde{y}$ (vector $\tilde{Y}$) denotes the measured value. Commands are noted u (vector U). We consider a discrete-time model of the form:

$$E : \left\{ \begin{array}{lcl} X(k+1) & = & f\big(X(k), U(k)\big) \\ Y(k) & = & g\big(X(k), U(k)\big) \\ 0 & \leq & h\big(X(k), U(k)\big) \end{array} \right. \qquad (1)$$

System A's behavior is described with rules of the form $\bigwedge_i e_i$ **if** ϕ, where $e_i \in E$ and ϕ is a conjunction of equalities/inequalities over functions of variables in $\mathcal{M} \cup \mathcal{X}$. A set $T = \{\tau_1, \cdots, \tau_{n_m}\}$ of transitions is specified for each mode m. Each transition τ is enabled according to a guard ϕ, and may trigger with probability $p(\tau)$ whenever the guard is satisfied. $T(s_i, s_j)$ denotes the set of transitions that moves A from s_i to s_j.

Given the ability A has to predict and diagnose its own behavior, we respectively note $\mathcal{P}(A)$ the prediction of the hybrid system's nominal state, and $\mathcal{D}(A)$ the diagnosis result after a fault occurs. Note that when fault modes are present, the diagnosis may become a state identification problem, and $\mathcal{P}(A)$, $\mathcal{D}(A)$ may result from the same engine. Uncertainty on the physical system's state requires to consider $\mathcal{P}(A)$ and $\mathcal{D}(A)$ as sets of hybrid states. We denote $\mathcal{S} = \big(\mathcal{P}(A), \mathcal{D}(A)\big)$.

Example (Pressure expansion system). *Figure 1 pictures our case study: a two valves system that limits water pressure between flow input Q_0 and flow output Q. An electric switch S powers valve V_2 when pressure P_0 equals or exceeds threshold P^*. V_2 opens when powered. S, V_1 and V_2 have two nominal operational modes* open *and* closed*, and two faulty modes* stuck_closed, stuck_open. *Q_0 and Q are measured. $P_0 \geq P_{atm}$ is the only input to the system. P_{atm} denotes the atmospheric pressure.*

Our scenario assumes faults occur when the prediction of the nominal state is uncertain[4], i.e. the uncertainty on the pressure does not allow to discriminate between two predicted states[5]:

$$s_N^1 : \left\{ \begin{array}{c} Q_0 > 0, P_0 < P^* \\ V_1.m = open \\ S.m = open \\ V_2.m = closed \\ Q_1 > 0, Q_2 = 0, Q > 0 \end{array} \right. \quad \text{and } s_N^2 : \left\{ \begin{array}{c} Q_0 > 0, P_0 \geq P^* \\ V_1.m = open \\ S.m = closed \\ V_2.m = open \\ Q_1 > 0, Q_2 > 0, Q > 0 \end{array} \right.$$

After observing $Q_0 > 0 \wedge Q = 0$, A returns diagnose, based on the knowledge of the nominal states above:

$$s_F^1 : \left\{ \begin{array}{c} Q_0 > 0, P_0 < P^* \\ \mathbf{V_1.m = stuck\_closed} \\ S.m = open \\ V_2.m = closed \\ Q_1 = 0, Q_2 = 0, Q = 0 \end{array} \right. , s_F^2 : \left\{ \begin{array}{c} Q_0 > 0, P_0 \geq P^* \\ \mathbf{V_1.m = stuck\_closed} \\ S.m = closed \\ \mathbf{V_2.m = stuck\_closed} \\ Q_1 = 0, Q_2 = 0, Q = 0 \end{array} \right.$$

$$\text{and } s_F^3 : \left\{ \begin{array}{c} Q_0 > 0, P_0 \geq P^* \\ \mathbf{V_1.m = stuck\_closed} \\ \mathbf{S.m = stuck\_open} \\ V_2.m = closed \\ Q_1 = 0, Q_2 = 0, Q = 0 \end{array} \right.$$

s_F^1 is the faulty state diagnosed from s_N^1 while s_F^2 and s_F^3 have been deduced from s_N^2. Hybrid states in $\mathcal{P}(A) = (s_N^1, s_N^2)$ and $\mathcal{D}(A) = (s_F^1, s_F^2, s_F^3)$ contain enough information for the autonomous system to extract its *functional deficiencies*.

3 Functional Deficiencies

Given a belief on a model-based system A, we extend $\mathcal{P}(A)$ and $\mathcal{D}(A)$ by the states probabilities such that $\mathcal{P}(A) = ((s_N^1, p(s_N^1)), \cdots, (s_N^n, p(s_N^n)))$ is the set of the n nominally predicted states, and their associated probabilities, and $\mathcal{D}(A) = ((s_F^1, p(s_F^1)), \cdots, (s_F^f, p(s_F^f)))$ the set of f faulty states from diagnosis, and their attached probabilities. Given a variable v, we note $s(v)$ its value in state s. Any set of nominal and faulty states in $\mathcal{S}$ is denoted a *reconfiguration set*. We want to find a set $\mathcal{F}$ of prioritized variable instances in $M \cup X$ that are the functional deficiencies between states in $\mathcal{P}(A)$ and $\mathcal{D}(A)$, and thus need to be recovered. The general idea that is developed in this section has been inspired by the model-based reconfiguration of logical functions in [13].

3.1 Deficient variable instances

Given two states (s_N, s_F) respectively from $\mathcal{P}(A)$ and $\mathcal{D}(A)$, and a variable v, we note $L\big(s_N(v), s_F(v)\big)$ the measure of the common ground of v's value in each state. We say that variable whose instances in a pair of nominal/faulty states have less common ground than observable variables that discriminate between these states, are deficient. We write that v is deficient if:

$$L\big(s_N(v), s_F(v)\big) \leq \frac{\sum_{y \in Y_{misb}} L\big(s_N(y), s_F(y)\big)}{nbr(Y_{misb})} \qquad (2)$$

where $nbr(Y_{misb})$ is the number of *misbehaving* observed variables. A misbehaving y is an observed variable that allowed the fault detection, thus discriminating s_N from s_F: y's value in s_F better fits $\tilde{y}$ than its value in s_N. When relation 2 is satisfied, we say $L\big(s_N(v), s_F(v)\big)$ is deficient. The expression of L and the misbehaving variables depend on the nature of the variables and the formalization of the uncertainty in the model.

[4] This corresponds to the general case of tracking multiple states simultaneously.

[5] Flows > 0 are abstracted from their real values for an improved readability.

In the case variable domains are discrete, as in [15], variable instances have attached boolean labels. Misbehaving variables are observables labeled 1 in s_N and 0 in s_F. We set up $L\big(s_N(v), s_F(v)\big) = 1 - \big(lab(s_N(v)) - lab(s_F(v)\big)$, where lab returns the label of a given instance. This case also applies to the measure of mode deficiencies.

In case variable instances are numerical intervals, as in [2], a misbehaving observed variable y is such that $s_N(y) \cap \tilde{y} = \emptyset$. We use $L\big(s_N(v), s_F(v)\big) = s_N(v) \cap s_F(v)$.

In case a variable estimate is represented with a Gaussian, as in [7], we say y is misbehaving if $p(\tilde{y} \mid s_F)p\big(T(s_N, s_F)\big) \geq p(\tilde{y} \mid s_N)$, i.e. if its likelihood is higher in the diagnosed estimate than in the nominally predicted one, given the probability of changing mode. Here $p\big(T(s_N, s_F)\big) = p\big(s_N(\phi_1, \cdots, \phi_r)\big) \prod_{i=1,\cdots,r} p(\tau_i)$ where r is the number of components, transiting from s_N to s_F. Given that $s_N \sim \mathcal{N}(m_N, \theta_N)$ and $s_F \sim \mathcal{N}(m_F, \theta_F)$, we define L as the measure of the common space enclosed by both density functions f_N, f_F. Given ρ^1, ρ^2 the two intersection points of these curves, and considering that $\theta_F \geq \theta_N$ (otherwise, the notations are inversed):

$$L\big(s_N(v), s_F(v)\big) = \int_{-\infty}^{\rho^1} f_N(v)dv + \int_{\rho^1}^{\rho^2} f_F(v)dv + \int_{\rho^2}^{+\infty} f_N(v)dv \quad (3)$$

ρ^1, ρ^2 are solutions of $f_N(v) = f_F(v)$. In the general case, at the curves intersection points, the Mahalanobis metric $(v-m)'\theta^{-1}(v-m)$ of both estimates is identical.

3.2 Functional Deficiencies

Based on deficient variables, we now build the functional deficiencies.

Definition 2 (Functional deficiency). *A functional deficiency F for a model-based system A over a set of hybrid states $\mathcal{S} = \big(\mathcal{P}(A), \mathcal{D}(A)\big)$ is a set of variable instances of $M \cup X$ that hold in some states of $\mathcal{P}(A)$, and that are* deficient *in some states of $\mathcal{D}(A)$. We denote as $S(F) \in \mathcal{S}$ the reconfiguration set associated to F.*

We write F as a conjunction of n_m mode instances and n_c mean value instances, $n_m + n_c = n$, as follows:

$$F = \bigwedge_{k=1,\cdots,n_m} C_k.m^{h_k} \bigwedge_{j=1,\cdots,n_c} \Big(\sum_{i=1,\cdots,p} p(s_N^i)s_N^i(v^j)\Big) \quad (4)$$

Then $(s_N^i, s_F^l) \in S(F)$ iff: $L\big(s_N^i(C_k.m^{h_k}), s_F^l(C^k.m^{h_k})\big)$ and $L\big(s_N^i(v^j), s_F^l(v^j)\big)$ are deficient for all i, j, k, l. In other words, $S(F)$ includes all nominal and faulty states whose pairs show a deficiency for all the instances of F. F is said to be *complete* w.r.t. a reconfiguration set S' iff $S' = S(F)$. The complete F over $\mathcal{S}$ is unique.

Property 1. *If F, F' are complete functional deficiencies, then if $F' \subseteq F$, $S(F) \subseteq S(F')$.*

Proof. If $F' \subseteq F$, then $S(F')$ contains at least all states of $S(F)$ as these show deficiencies for all instances of F, plus potential states that do not show deficiencies for instances in $F \setminus F'$. □

Property 2. *If F, F' are complete functional deficiencies and $S(F) = S(F')$, then $F = F'$.*

Proof. This comes from the uniqueness of a complete functional deficiency over a given reconfiguration set S. □

Given two tuples $\big(F_1, S(F_1)\big)$ and $\big(F_2, S(F_2)\big)$, we write:

$$\big(F_1, S(F_1)\big) \cap \big(F_2, S(F_2)\big) = \big(F_1 \cap F_2, S(F_1) \cup S(F_2)\big) \quad (5)$$

$$\big(F_1, S(F_1)\big) \cup \big(F_2, S(F_2)\big) = \big(F_1 \cup F_2, S(F_1) \cap S(F_2)\big) \quad (6)$$

We note $F_1 \cap F_2$, $F_1 \cup F_2$ for short. From now on we consider a functional deficiency to be complete when not explicitly mentioned otherwise. Also, we sometimes write a functional deficiency as the conjunction of its elements. The tuple $\big(F, S(F)\big)$ is denoted a *reconfiguration tuple*. Finally, it is possible to prioritize[6] a functional deficiency:

$$pr(F) = \sum_{i=1}^{n} \sum_{j=1}^{f} p(s_N^i)p(s_F^j),\ (s_N^i, s_F^j) \in S(F) \quad (7)$$

Definition 3 (Core functional deficiency). *The core functional deficiency F^c has its elements satisfied in* all *states of $\mathcal{P}(A)$ and* deficient *in* all *states of $\mathcal{D}(A)$. F^c is unique for a given set $\mathcal{S}$, and its priority is equal to* 1.[7]

Note that at least all misbehaving variables in states of $S(F)$ do belong to the core deficiency, as does $Q = 0$ in our example.

3.3 Minimal functionalities over maximal reconfiguration sets

This section develops a characterization of functional deficiencies whose size is minimal, while deficient over the largest number of state estimates. The reason behind this effort is that the autonomous system certainly wants to operate minimal changes while covering the maximum states. From properties 1 and 2, the reconfiguration set increases in size when the functionality decreases in size. A complete functional deficiency of minimal size over a maximal reconfiguration set is then easily characterized.

Definition 4 (Minimal functional deficiency over the maximal reconfiguration set). *A minimal functional deficiency F has a maximal reconfiguration set $S(F)$ if it exists no other functional deficiency F' such that $S(F) \subset S(F')$ and $F' \subset F$.*

The search for *minimal* functional deficiencies over *maximal* reconfiguration sets leads to a set of functional deficiencies denoted *minimax. A minimax functional deficiency represents the minimum set of variable instances that are deficient over the same maximum set of pairs of nominal/faulty states.*

Proposition 1. *Given two minimax functional deficiencies F and F' such that $F' \cap F \neq \emptyset$, then $S(F') = S(F)$.*

Proof. If $F'' = F' \cap F$ and $F'' \neq \emptyset$, then if $F'' \subset F$, from definition 4 and property 1, it comes $S(F) = S(F'')$. Similarly, $F'' \subset F'$ yields $S(F'') = S(F')$, so $S(F) = S(F')$. The same result is obtained if $F'' = F$ or $F'' = F'$ with property 2. □

[6] Note that in this expression, there is no notion of fault criticality. Every faulty state is assumed to have equal criticality but the probability of the state is taken into account.

[7] Given that $\mathcal{P}(A)$ and $\mathcal{D}(\mathcal{A})$ have their state probabilities summing to 1.

The previous proposition implicitly focuses the search on *distinct minimax functionalities*. Thus functional deficiencies may be characterized as disjoint sets of variable instances. This result brings flexibility to the reconfiguration process under uncertainty, but is mitigated as the disjoint functions are not independent from each other w.r.t. to the hybrid dynamics. In other words, they may not be recovered independently. In reference to the recovery (planning) operation, these functionalities are no serializable goals.

Proposition 2. *The core functional deficiency F^c is minimax.*

Proof. This is trivial from definition 4. F^c is also complete with $S(F^c) = \mathcal{S}$. □

3.4 Functional Deficiencies Computation

```
1: Compute the complete $F$ w.r.t. each reconfiguration set $(s_N^p, s_F^q)$, compute $F^c$, and add them all to the agenda.
2: Iterate through the tuples $(F_i, F_j)$ in the agenda.
3: If $F^c \cap F_i \neq \emptyset$, $F_i \longleftarrow F_i \setminus \{F_i \cap F^c\}$.
4: Else if $F_i \cap F_j \neq \emptyset$, create a new function $F' = F_i \cap F_j$ and add it to the agenda. Do $F_i \longleftarrow F_i \setminus F'$.
5: Else if $F_i = F_j$, $S(F_i) = S(F_i) \cup S(F_j)$ and remove the remaining function $F_j$ from the agenda.
6: $F_i$ is minimax when it does not intersect with other functions anymore. It is removed to the agenda and returned.
```

Algorithm 1: Computing minimax functional deficiencies

The computation of the minimax functional deficiencies is performed with algorithm 1. Its main principle is to progressively reduce simple complete, but non minimax deficiencies. The first step updates the deficiencies for each combination of two states of $\mathcal{S}$ using the measure of relation 2, and computes the core function. Iterating through this set, step 3 prunes out any deficiency of its intersection with F^c. Step 4 prunes out non-disjoints functionalities of their intersection and creates a new deficiency with it. Step 5 merges the reconfiguration sets of similar deficiencies.

The algorithm is better understood by developing our example. Step 1 gives:

$$\begin{array}{rcl}
s_N^1, s_F^1 & : & F_1 = (V_1.m = open) \wedge Q_1 > 0 \wedge Q > 0 \\
s_N^1, s_F^2 & : & F_2 = P_0 < P^* \wedge (S.m = open) \\
 & & \wedge (V_2.m = closed) \wedge Q_1 > 0 \wedge Q > 0 \\
 & & \wedge (V_1.m = open) \\
s_N^1, s_F^3 & : & F_3 = P_0 < P^* \wedge (S.m = open) \\
 & & \wedge Q_1 > 0 \wedge Q > 0 \wedge (V_1.m = open) \\
s_N^2, s_F^1 & : & F_4 = P_0 \geq P^* \wedge (S.m = closed) \\
 & & \wedge (V_1.m = open) \wedge (V_2.m = open) \\
 & & \wedge Q_1 > 0 \wedge Q_2 > 0 \wedge Q > 0 \\
s_N^2, s_F^2 & : & F_5 = (V_1.m = open) \wedge (V_2.m = open) \\
 & & \wedge Q_1 > 0 \wedge Q_2 > 0 \wedge Q > 0 \\
s_N^2, s_F^3 & : & F_6 = (S.m = closed) \wedge (V_1.m = open) \\
 & & \wedge Q_1 > 0 \wedge Q_2 > 0 \wedge Q > 0 \\
 & & \wedge (V_2.m = open) \\
s_N^1, s_N^2, s_F^1, s_F^2, s_F^3 & : & F^c = (V_1.m = open) \wedge Q_1 > 0 \wedge Q > 0
\end{array}$$

We have $F_1 = F^c$ so F_1 can be eliminated. Then reducing other functions with F^c:

$$\begin{array}{rcl}
F_2 & = & P_0 < P^* \wedge (S.m = open) \wedge (V_2.m = closed) \\
F_3 & = & P_0 < P^* \wedge (S.m = open) \\
F_4 & = & P_0 \geq P^* \wedge (S.m = closed) \wedge (V_2.m = open) \wedge Q_2 > 0 \\
F_5 & = & (V_2.m = open) \wedge Q_2 > 0 \\
F_6 & = & (S.m = closed) \wedge Q_2 > 0 \wedge (V_2.m = open)
\end{array}$$

1. $F_2 \cap F_3 = P_0 < P^* \wedge (S.m = open)$, $F_7 \longleftarrow P_0 < P^* \wedge (S.m = open)$, $S(F_7) = (s_N^1; s_F^2, s_F^3)$, $F_2 = F_2 \setminus F_7 = (V_2.m = closed)$, $S(F_2) = (s_N^1; s_F^2)$. F_7 is added to the agenda.
2. $F_2 \cap F_4 = \emptyset$, $F_2 \cap F_5 = \emptyset$, $F_2 \cap F_6 = \emptyset$, and $F_2 = V_2.m = closed$ is minimax.
3. $F_3 \cap F_4 = \emptyset$, $F_3 \cap F_5 = \emptyset$, $F_3 \cap F_6 = \emptyset$, $F_3 = F_7$, remove F_7, $S(F_3) = (s_N^1; s_F^2, s_F^3)$. $F_3 = P_0 < P^* \wedge (S.m = open)$ is minimax.
4. $F_4 \cap F_5 = F_5$, $F_4 \longleftarrow F_4 \setminus F_5 = P_0 \geq P^* \wedge (S.m = closed)$, $S(F_4) = (s_N^2; s_F^1)$. $S(F^5) = (s_N^2; s_F^1, s_F^2)$.
5. $F_4 \cap F_6 = (S.m = closed)$, $F_8 = (S.m = closed)$, $S(F_8) = (s_N^2; s_F^1, s_F^3)$, $F_4 \longleftarrow F_4 \setminus F_8 = P_0 \geq P^*$, $S(F_4) = (s_N^2; s_F^1)$, and F_4 is minimax.
6. $F_6 \cap F_5 = F_5$, $F_6 \longleftarrow F_6 \setminus F_5 = F_8$. Remove F_8, $F_6 = (S.m = closed)$, $S(F_6) = (s_N^2; s_F^1, s_F^3)$. F_5, F_6 are minimax. $S(F^5) = (s_N^2; s_F^1, s_F^2, s_F^3)$.

Finally, the minimax functions are:

$$\begin{array}{l}
F^c = (V_1.m = open) \wedge Q_1 > 0 \wedge Q > 0 \,,\, S(F^c) = (s_N^1, s_N^2; s_F^1, s_F^2, s_F^3) \\
F_2 = (V_2.m = closed) \,,\, S(F_2) = (s_N^1; s_F^2) \\
F_3 = P_0 < P^* \wedge (S.m = open) \,,\, S(F_3) = (s_N^1; s_F^2, s_F^3) \\
F_4 = P_0 \geq P^* \,,\, S(F_4) = (s_N^2; s_F^1) \\
F_5 = (V_2.m = open) \wedge Q_2 > 0 \,,\, S(F_5) = (s_N^2; s_F^1, s_F^2, s_F^3) \\
F_6 = (S.m = closed) \,,\, S(F_6) = (s_N^2; s_F^1, s_F^3)
\end{array}$$

At this point, a possible extension to the functional deficiencies is to distinguish the *continuous reduction* of F_i, that is its reduction to variables in $\mathcal{X}$, from the *hybrid* deficiency (made of both discrete and continuous instances). Intuitively, as the modes are relaxed, there exist more states that satisfy the continuous reduction to a deficiency, than the hybrid deficiency. For this reason, we say the latter leads to *reset* solutions (as modes deficiencies are explicitly set up to be recovered), as opposed to *redundancy* solutions (modes are unspecified, several component modes may recover the continuous deficiencies). We note $\bar{F}$ the continuous reduction to F.

4 Reconfiguration of Functional Deficiencies

This section focuses on reconfiguring a functional deficiency by identifying a set of goal states, and planning a recovery to those states. Ideally, a goal state specifies a value to all component modes, and may be inferred from a functional deficiency. In the case of a hybrid uncertain state however, the constraints in the form of continuous static/differential equations prevent a unique identification of the modes from a given continuous state point. Hence we propose to rely on an intrinsic property of hybrid systems, that is that the conditional statements ϕ naturally partition their behavioral space into hybrid regions that we refer to as *configurations*. We refer the reader to [2] for a formalization of these regions.

In the following, we denote as the *goal functional deficiency* F^* the functional deficiency to be recovered. Its selection is part of the recovery process, and is detailed at the end of the section. For now, we pick up a simple F^* as F^c because its priority is maximal, and it covers all state estimates.

Identifying the hybrid regions that enclose the values of F^* is sufficient as to form goals that we refer to as *configuration goals* (instead of goal states). They correspond to reduced sets of both component modes and equalities/inequalities over continuous variables.

Then, we must ensure that the goals are reachable by both the continuous and discrete dynamics, respectively equations E and transitions T.

4.1 Configurations identification

Here, we determine the goal configurations through a process similar to the model-based diagnosis consistency approach. Indeed, reconfiguration can be viewed as the problem of identifying components whose reconfiguration is sufficient to restore acceptable behavior, when diagnosis is the problem of identifying components whose abnormality is sufficient to explain observed malfunctions [4].

4.1.1 Causal-graph of influences

A first difficulty lies in equations in E that may demand a time-analysis for determining continuous variable values that are not set in F^*. A second problem lies in the non-existence of a bijection between modes M and a particular continuous region of the state-space, as constrained by E. These problems can be tackled by first enhancing the model-based formalism with a causal representation of E.

Definition 5 (Causal-Graph of Influences). *The causal-graph of influences of a set of equations E is an oriented graph $G = (X, I)$ where the variables in X form a set of nodes x_i, and I a set of arcs among these variables.*

The causal-graph is a representation of relations among variables in E that holds at any time step.

Definition 6 (Causal Influence). *A causal influence in I, $I_{i,j} = (x_i, x_j, b, \phi)$, is a directed arc between two variables x_i and x_j, with b the* sign *of the influence and ϕ its* activation *condition.*

Influences are drawn from the implicit causality in E. Variables that are subject to no influence are referred to as the *inputs* of G. Figure 2 pictures the causal-graph of the pressure expansion system. In the following we replace equations in E with G.

In general some work is required to extract the causality from static relations [14]. $b = \{-1, 1\}$ stores the numerical *positive or equal*/*negative* influence among variables. ϕ's truth value in the hybrid state determines the *activation*/*deactivation* of the influence in the graph. Unconditioned, the influence is permanently activated. The activation conditions represent the causality changes in the dynamics.

Definition 7 (Configuration). *A configuration for G (and by extension A) is of the form $\bigwedge_i \phi_i$.*

A configuration delimits a region of behavior of A. In our example, $V_1.m = open \wedge V_2.m = open \wedge P_0 \geq P^* \wedge P_0 \geq P_1 \wedge P_0 \geq P_2 \wedge S.m = closed$ is a nominal configuration of the system.

4.1.2 Building configuration goals from functional deficiencies

We write the MBD theory based on consistency [12] where for the reconfiguration purpose, observations are replaced with functional deficiencies. A deficiency F^* has been characterized w.r.t. the state uncertainty. We are now searching for the *minimal sets of conditions* that are sufficient to restore F^*.

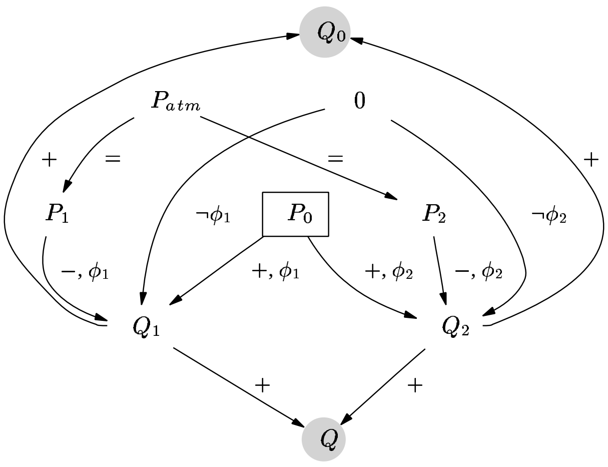


Figure 2. Pressure expansion system causal-graph

Definition 8 (Reconfiguration candidate). *A reconfiguration candidate for A given F^* is defined as a minimal set $\Delta = \{I_1^\Delta, \cdots, I_h^\Delta\} \subseteq I$ of influences such that*

$$A \cup F^* \cup \neg\phi_1^\Delta \cup \cdots \cup \neg\phi_h^\Delta \tag{8}$$

is consistent.

Definition 9 (Reconfiguration conflict). *A reconfiguration conflict for A given F^* is a set $\lambda = \{I_1^c, \cdots, I_k^c\}$ of influences such that*

$$A \cup F^* \cup \phi_1^c \cup \cdots \cup \phi_k^c \tag{9}$$

is not consistent.

From $G \cup F^*$, we seek for reconfiguration conflicts in G that are such that influences in a conflict cannot be activated together given F^*. For a deficient variable (node) x_j of F^*, we call *ascending* influences the influences that belong to the paths from the inputs/other deficient variables, to x_j. An ascending influence I_i for x_j is noted $\lambda_i^j = \{I_i, \phi_i\}$. A conflict for x_j is thus the set λ^j of its ascending influences $\{\lambda_i^j\}_{i=1,\cdots,n_j}$. $\Lambda = \left\{\{\lambda^j\}_{j=1,\cdots,n_{F^*}}\right\}$ is the collection of conflicts over all deficient variables of F^*. The minimal set of influences Δ that are candidates to the reconfiguration is obtained similarly to the diagnoses in the MBD theory by computing the hitting sets (HS) over Λ [12]. We note $\Delta_q = (\mathcal{I}_q, \wedge_{I_i \in \mathcal{I}_q} \phi_i)$ a diagnostic candidate, where $\mathcal{I}_q$ is a set of influences. Consequently, $\Delta = \left\{\{\Delta_q\}_{q=1,\cdots,n_q}\right\}$. We note $\neg\Delta = \left\{\{\neg\Delta_q\}_{q=1,\cdots,n_q}\right\}$.

```
1: Apply $F^*$ to $G$.
2: Apply $S_F(F^*)$ to $G \setminus F^*$.
3: Get the conflicts $\Lambda$.
4: Compute $\Delta = HS(\Lambda)$.
5: $\neg\Delta \wedge F^*$ are goal configurations.
```

Algorithm 2: Identifying reconfiguration candidates ($Goals$)

Consider our example again. Reconfiguring $F^* = F^c$ with algorithm 2 implies ϕ_1 is satisfied (step 1), and based on remaining variable instances in states in $S_F(F^*)$ the configuration of the subgraph $G \setminus F^*$ (G deprived of nodes and axis to nodes in F^*) is determined, in that case $\neg\phi_2$ is satisfied (step 2). Tracing the ascending influences in G, it comes two sets of conflicts (one per continuous variable instance in F^*):

$$\begin{cases} \lambda_Q = \{Q \leftarrow Q_1, Q \leftarrow Q_2, Q_2 \overset{\neg\phi_2}{\leftarrow} 0, P_2 \leftarrow P_{atm}\} \\ \lambda_{Q_1} = \{Q_1 \overset{\phi_1}{\leftarrow} P_0, Q_1 \overset{\phi_1}{\leftarrow} P_1, P_1 \leftarrow P_{atm}\} \end{cases}$$

ϕ_1 is satisfied in F^c, and influences over Q, P_1 and P_2 are activated in all configurations, so it simplifies to:

$$\begin{cases} \lambda_Q = \{Q_2 \overset{\neg\phi_2}{\leftarrow} 0\} \\ \lambda_{Q_1} = \{\} \end{cases}, \Lambda = \{\lambda_Q, \lambda_{Q_1}\}$$

It comes $\Delta = \big\{\{\neg\phi_2\}\big\}$ and $\phi_2 \wedge F^c$ thus is a valid goal configuration (step 5). Reconfiguring the continuous reduction $\bar{F}^c$ leads to more opportunities: ϕ_1 is no more satisfied and $\lambda_{Q_1} = \{\neg\phi_1\}$, thus $\Delta = \{\{\neg\phi_1, \neg\phi_2\}\}$ and configuration goals are given by $\phi_1 \wedge \phi_2 \wedge \bar{F}^c$.

4.2 Recovery

The recovery operation aims at bringing the system into the regions defined by the configuration goals. Due to the hybrid dynamics, a solution is a chain of transitions to the component mode goals, while the continuous dynamics ensure the transition guards are successively satisfied. Solution sets of component transitions $T_0, \cdots, T_p$ must satisfy

$$A \cup \mathcal{D}(A) \cup T_0 \cup \cdots \cup T_p \cup F^* \cup \neg\Delta \tag{10}$$

is consistent, where the current time of the system is set to k_0 and the initial state belongs to $\mathcal{D}(A)$. $Pl = \{T_0, \cdots, T_p\}$ is a *plan* for the recovery. Noting k_p the time at which transition T_p triggers, the continuous dynamics must satisfy

$$\begin{cases} X(k_0) \cup \phi_0 \\ E(X(k_0)) \cup \phi_1 \\ E(X(k_1)) \cup \phi_2 \\ \vdots \\ E(X(k_{p-1})) \cup \phi_p \\ E(X(k_p)) \cup F^* \end{cases} \tag{11}$$

are consistent, where $E(X(k_j))$ refers to the dynamics of relation (1), is conditioned by ϕ_{j+1}, and $X(0) = \sum_{s_F^i \in \mathcal{D}(A)} p(s_F^i) X_F^i(0)$. We say relations (10) and (11) define a *hybrid system planning* problem. To our knowledge, the planning of hybrid systems has received no attention yet. We believe that several control and planning problems may be casted into this formalism.

Relation (10) defines a probabilistic conformant planning problem [8], where a set of transitions must bring the system to a set of predetermined goals, under uncertainty and without observing the system full state. The plan maximizes the probability of the goal configuration given the initial belief state $\mathcal{D}(A)$. In our example, a stuck valve cannot be re-opened, so no plan exists for functionalities F^c and $\bar{F}^c$. A plan exists to F_5 for some initial states, $Pl = \{\tau_3, \tau_{21}\}$. F_6 has a plan $Pl = \{\tau_3\}$.

Relation (11) defines a control problem where the continuous dynamics must be forced to successive ϕ_j through available inputs. A model predictive control problem (MPC) solves on-line a finite horizon open-loop optimal control problem subject to system dynamics and constraints involving states and controls. Based on measurements obtained at time k, the future dynamic behavior of the system is predicted over a fixed horizon, and the controller determines the input such that a performance criterion is optimized. This technique fits well within the model-based autonomous system framework, given that two key elements are already present, the model A, and the state predictor (or estimator) $\mathcal{P}(A)$. By using control and measurement horizons of a single time step, a basic formulation of the MPC problem at time k is

$$\begin{aligned} U^*(k+1) &= \min_U J(X(k), U(k)) \\ J(X(k), U(k)) &= \int_k^{k+1} F(X(t), U(t))dt \\ F(X, U) &= (X - X_s)^T Q(X - X_s) \\ &\quad +(U - U_s)^T R(U - U_s) \\ X(k+1) &= f(X(k), U^*(k)) \\ 0 &\leq h(X(k), U(k)) \end{aligned}$$

where Q and R denote positive definite symmetric weighting matrices, and $U^*(k+1)$ is the optimal input used in the prediction at $k+1$. Considering ϕ over X in the form ϕ: $l(X) \geq 0$, we note $\bar{\phi}$: $\bar{l}(X) + \epsilon = 0$ its reduction to an equality, where ϵ is a term that ensures the threshold is later satisfied. The function is evaluated at k with $\bar{\phi}(k)$: $\bar{l}(X(k)) + \epsilon$, and we note its inverse $\bar{\phi}^{-1}(k)$. The MPC application to the control objective ϕ_j sets the setting point (X_s, U_s) to $(\bar{\phi}_j^{-1}(k), 0)$. In our example, τ_3's guard gives $\bar{\phi}_{\tau_3}^{-1}(k) = P^* + \epsilon'$.

Again, we face the fact that $\mathcal{P}(A)(k) = \{s^1, \cdots, s^q\}$ likely contains multiple state estimates. Thus the minimization must apply to each $F(X^i(k), U(k))$, returning $U^{*,i}(k+1)$. We merge the optimized input candidates according to the states estimated probabilities:

$$U^*(k+1) = \sum_{i=1,\cdots,q} p(X^i(k)) U^{*,i}(k+1) \tag{12}$$

Finally, when ϕ_j is reached, transition T_j should trigger, and MPC then focuses on ϕ_{j+1}. The last MPC set-point is F^*.

Solving this control problem for complex system however requires more research. First, the MPC community itself seeks for better integration of modern state estimation techniques within the control loop [9]. Second, ϕ's inverse is a problem in practice. The control could focus on bringing the system state back to the geometrical center of the goal configuration region instead. This is yet to be explored. Third, optimality and especially, stability problems, if far out of the scope of this paper, must be tackled in the case of control based on multiple state estimates. Finally, it is likely that modern hybrid state estimators are coupled with more powerful techniques such as Quasi-Infinite Horizon NMPC [3]. Note that recent developments also pave the way for stability and safety/reachability analysis of these controllers [1].

4.3 Reaching the goals: safety and convergence

Considering the context of a faulty system, the reconfiguration process should likely be safe, not making the situation worse. In our case, the goal configurations identification may produce multiple solutions, while not ensuring that they are reachable. In this section we improve algorithm 2 by reducing the number of goal solutions that are guaranteed to be reachable under monotonous continuous dynamics. To ensure the latter, and given a variable v that appears in F^* (instance v^*), the sign of $(S_N(v) - S_F(v))$ is studied, where $S(F^*) = (S_N, S_F)$. Here, we use $S_N(v) - S_F(v) = v^* - \sum_{s_F^i \in \mathcal{D}(A)} p(s_F^i) s_F^i(v)$. Algorithm 2 is modified such that Λ

becomes Λ^-, *the set of influences to be deactivated*, while Λ^+, *the set of influences to be activated* is constructed as follows:

- Given a path of ascending influences $\{I_{i,i_1}, \cdots, I_{i_n,j}\}$ from x_i to x_j involved in F^*, if $x_i\big(S_N(x_j) - S_F(x_j)\big) \prod_{k=i_1,\cdots,i_n} b_k > 0$, then for each ϕ_k that is not satisfied, add $I_{i_k,i_{k+1}}$ to Λ^+.
- Otherwise, if the above criterion is not satisfied, while ϕ_k is, then add $I_{i_k,i_{k+1}}$ to Λ^-.

This corresponds to activating every ascendant path whose combined influences have a beneficial effect to the restoration of F^*.

```
1: Apply $F^*$ to $G$.
2: Apply $S_F(F^*)$ to $G \setminus F^*$.
3: Get the conflicts $\Lambda^+$, $\Lambda^-$.
4: Compute $\Delta^+ = HS(\Lambda^+)$ and $\Delta^- = HS(\Lambda^-)$.
5: Do $\Delta = \Delta^+ \bigotimes \neg\Delta^-$ and eliminate inconsistent configurations.
6: $\Delta \wedge F^*$ are goal configurations.
```

Algorithm 3: Identifying reconfiguration candidates ($SafeGoals$)

Back to our example, we reconfigure $\bar{F}_5 = Q_2 > 0$. Step 3 of algorithm 3 gives $\lambda^+_{Q_2} = \{Q_2 \overset{\phi_2}{\leftarrow} P_0\}$, $\lambda^-_{Q_2} = \{Q_2 \overset{\neg\phi_2}{\leftarrow} 0\}$, thus $\Delta^+ = \{\{\phi_2\}\}$, $\Delta^- = \{\{\neg\phi_2\}\}$. The solution is the same as returned by algorithm 2 but it is now ensured that opening V_2 brings the flow back into the right direction.

The safety may not be ensured when negative and positive effects to a variable are activated via the same condition, as over Q_2 in our example. If P_{atm} was not considered being a constant, a numerical analysis would have been required here.

4.4 Prioritized selection of functional deficiencies

Our general strategy to the reconfiguration of the functional deficiencies explores *reset* solutions first, then *redundancy* solutions (continuous reductions) in prioritized order. In case of plan failure the next deficiency is selected (algorithm 4). In our example, s_F^2 and s_F^3 have much lower probability than s_F^1 as they correspond to double faults. F^c is subject to plan failure. F_6: $S.m = closed$ is its own goal configuration and has a plan τ_3 whose guard is $P_0 \geq P^*$. MPC generates the pressure input P_0 to reach that level. Note that depending on the real initial state, the reconfiguration may have no effect. The operation does not harm the system as we consider that maintaining a nominal level of pressure does not harm even the faulty system, and may help discriminate among the estimates. For example, if reconfiguring F_6 fails, s_F^1, and potentially s_F^2 are eliminated.

```
1: Compute functional deficiencies with algorithm 1
2: Identify goal configurations with algorithm 2 or 3.
3: Find a plan, in case of failure move to the next deficiency, in
   prioritized order.
4: Apply MPC using $\mathcal{P}(A)$ as the predictor.
```

Algorithm 4: Prioritized selection of functional deficiencies

5 Summary, Existing works and Perspectives

We've presented a methodology to the automated reconfiguration of functional deficiencies. The deficiencies are identified by comparing predicted and diagnosed states, and then partitioned and prioritized over the state estimates. Goals are further identified from the deficiencies. Planning and MPC techniques are used in common to move the system toward the goals.

To our knowledge, automated MBReconf has not yet received much attention. A pioneer work, [4], explores the analogy between the problems of diagnosis and reconfiguration. [13] examines the use of diagnosis for the reconfiguration and develops logical functionalities. Goal identification and safe planning have been studied in [16] in the case of qualitative models. We are not aware of any work about the planning of hybrid systems.

Several improvements are planned. First, it appears that restoring a single minimax deficiency does not restore a full nominal state: an alternate strategy would be to combine the deficiencies so to restore a single nominal state that would be selected to maximize the chances of a successful reconfiguration w.r.t. the uncertainty on the faulty estimate. Second, the $SafeGoals$ algorithm should be enhanced to tackle more complex dynamics. Third, we would like to explore and formalize the planning of hybrid systems.

REFERENCES

[1] A. Bemporad, W.P.M.H. Heemels, and B. De Schutter, 'On hybrid systems and closed-loop mpc systems', in *Proceedings of the 40th IEEE Conference on Decision and Control, Orlando, Florida, USA*, (December 2001).

[2] E. Benazera and L. Travé-Massuyès, 'The consistency approach to the on-line prediction of hybrid system configurations', in *Proceedings of the IFAC Conference on Analysis and Design of Hybrid Systems 2003*, (2003).

[3] H. Chen and F. Allgwer, 'A quasi-infinite horizon nonlinear model predictive control scheme with guaranteed stability', *Automatica*, **34**(10), (1998).

[4] J. Crow and J. Rushby, 'Model-based reconfiguration: toward an integration with diagnosis', in *Proceedings of AAAI-91, Anaheim, CA*, volume 2, pp. 836–841, (1991).

[5] W. Hamscher, L. Console, and J. De Kleer, *Readings in Model-Based Diagnosis*, Morgan Kaufmann, San Mateo, CA, 1992.

[6] M. Hofbaur and B.C. Williams, 'Mode estimation of probabilistic hybrid systems', *Hybrid Systems: Computation and Control, Lecture Notes in Computer Science (HSCC 2002)*, **2289**, 253–266, (2002).

[7] F. Hutter and R. Dearden, 'The gaussian particle filter for diagnosis of non-linear systems', in *Proceedings of the Thirteenth International Workshop on Principles of Diagnosis DX-03*, (2003).

[8] N. Hyafil and F. Bacchus, 'Conformant probabilistic planning via csps', in *Proceedings of the Thirteenth International Conference on Automated Planning and Scheduling (ICAPS 03)*, (2003).

[9] M. Morari and J. H. Lee, 'Model predictive control: past, present, future', in *Joint 6th International Symposium on Process Systems Engineering (PSE'97)*, (1997).

[10] N. Muscettola, P. Pandurang Nayak, Brian C. Williams, and B. Pell, 'Remote agent : To boldly go where no ai system has gone before', *Artificial Intelligence*, **103**, 5–47, (1998).

[11] P. Nayak and J. Kurien, 'Back to the future for consistency-based trajectory tracking', in *Proceedings of AAAI-2000, Austin, Texas*, (2000).

[12] R. Reiter, 'A theory of diagnosis from first principles', *Artificial Intelligence*, (32), 57–95, (1987).

[13] M. Stumptner and F. Wotawa, 'Reconfiguration using model-based diagnosis', in *Proceedings of the Tenth International Workshop on Principles of Diagnosis DX-99*, (1999).

[14] L. Travé-Massuyès and R. Pons, 'Causal ordering for multiple modes systems', in *Proceedings of the Eleventh International Workshop on Qualitative Reasoning*, pp. 203 – 214, (1997).

[15] B. C. Williams and P. Nayak, 'A model-based approach to reactive self-configuring systems', in *Proceedings of AAAI-96, Portland, Oregon*, pp. 971–978, (1996).

[16] B. C. Williams and P. Nayak, 'A reactive planner for a model-based executive', in *Proceedings of IJCAI-97*, (1997).